Automata and Formal Languages (4)

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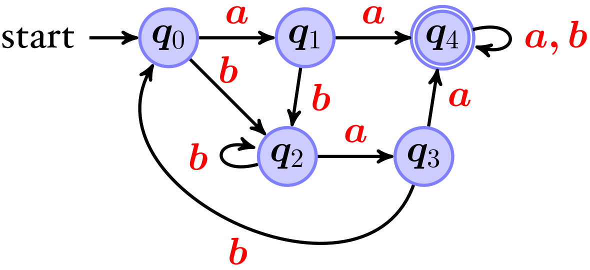
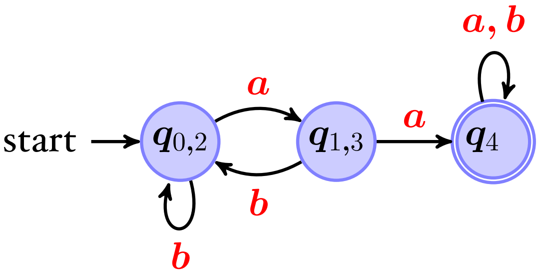
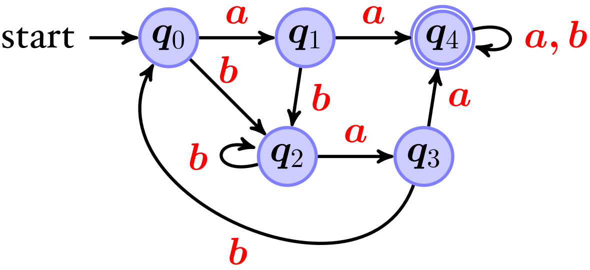
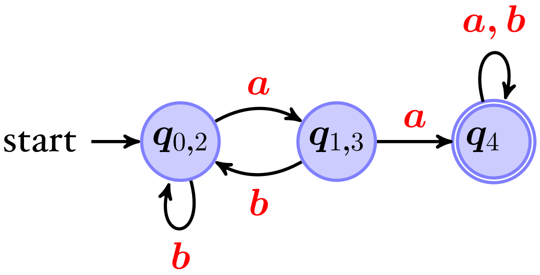
Slides: KEATS (also home work is there)





minimal automaton





minimal automaton

1. Take all pairs (q, p) with $q \neq p$
2. Mark all pairs that are accepting and non-accepting states
3. For all unmarked pairs (q, p) and all characters c tests wether

$$(\delta(q,c), \delta(p,c))$$

   are marked. If yes, then also mark (q, p)
4. Repeat last step until no chance.
5. All unmarked pairs can be merged.

Last Week

Last week I showed you

- a tokenizer taking a list of regular expressions
- tokenization identifies lexeme in an input stream of characters (or string) and cathegorizes them into tokens

Two Rules

- Longest match rule (maximal munch rule): The longest initial substring matched by any regular expression is taken as next token.

- Rule priority: For a particular longest initial substring, the first regular expression that can match determines the token.

"if true then then 42 else +"

KEYWORD:
"if", "then", "else",
WHITESPACE:
" ", "\n",
IDENT:
LETTER · (LETTER + DIGIT + "_")*
NUM:
(NONZERODIGIT · DIGIT*) + "0"
OP:
"+"
COMMENT:
"/*" · (ALL* · "*/" · ALL*) · "*/"

"if true then then 42 else +"

KEYWORD(if),
WHITESPACE,
IDENT(true),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
KEYWORD(then),
WHITESPACE,
NUM(42),
WHITESPACE,
KEYWORD(else),
WHITESPACE,
OP(+)

"if true then then 42 else +"

KEYWORD(if),
IDENT(true),
KEYWORD(then),
KEYWORD(then),
NUM(42),
KEYWORD(else),
OP(+)

There is one small problem with the tokenizer. How should we tokenize:

$$\text{"x - 3"}$$

OP:

"+", "-"

NUM:

(NONZERODIGIT · DIGIT*) + "0"

NUMBER:

NUM + ("-" · NUM)

Negation

Assume you have an alphabet consisting of the letters a, b and c only. Find a regular expression that matches all strings except ab, ac and cba.

Deterministic Finite Autom

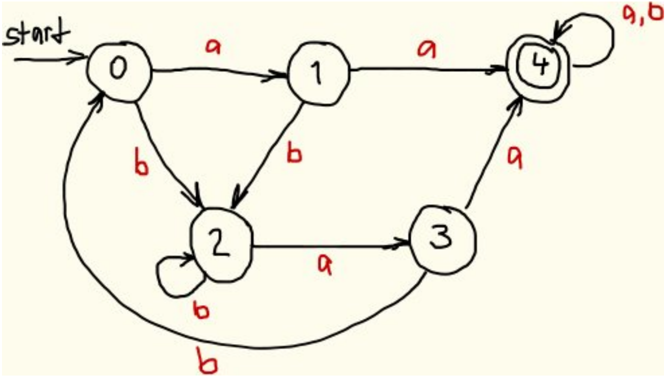
A deterministic finite automaton consists of:

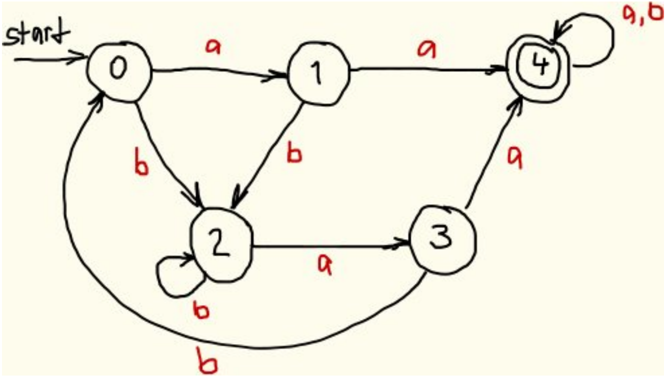
- a finite set of states
- one of these states is the start state
- some states are accepting states, and
- there is transition function

  which takes a state and a character as arguments and produces a new state

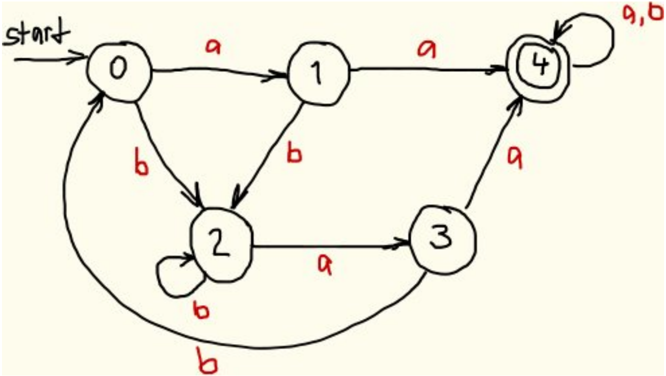
  this function might not always be defined everywhere

$$A(Q, q_0, F, \delta)$$


start
0
1
2
3
4
a
a
a
a
a,b
b
b
b
b



- start can be an accepting state
- it is possible that there is no accepting state
- all states might be accepting (but does not necessarily mean all strings are accepted)



for this automaton δ is the function

$$
\begin{array}{lll}
(q_0, a) \rightarrow q_1 & (q_1, a) \rightarrow q_4 & (q_4, a) \rightarrow q_4 \\
(q_0, b) \rightarrow q_2 & (q_1, b) \rightarrow q_2 & (q_4, b) \rightarrow q_4
\end{array} \quad \ldots
$$

Accepting a String

Given

$$A(Q, q_0, F, \delta)$$

you can define

$$\begin{aligned} \hat{\delta}(q, \text{""}) &= q \\ \hat{\delta}(q, c :: s) &= \hat{\delta}(\delta(q, c), s) \end{aligned}$$

Accepting a String

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$$A(Q, q_0, F, \delta)$$

you can define

$$\begin{aligned} \hat{\delta}(q, \text{""}) &= q \\ \hat{\delta}(q, c :: s) &= \hat{\delta}(\delta(q, c), s) \end{aligned}$$

Whether a string s is accepted by A?

$$\hat{\delta}(q_0, s) \in F$$

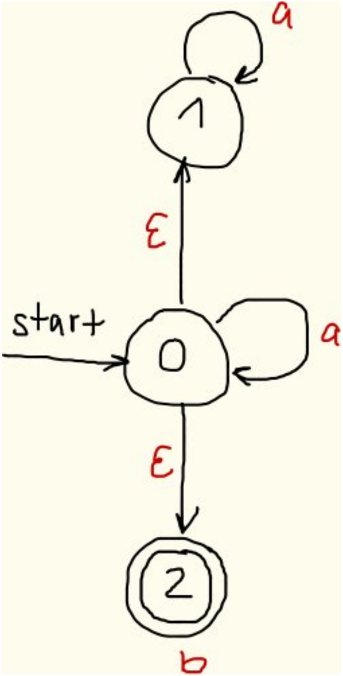
Non-Deterministic Finite Automata

A non-deterministic finite automaton consists again of:

- a finite set of states
- one of these states is the start state
- some states are accepting states, and
- there is transition relation

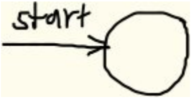
$$(q_1, a) \rightarrow q_2$$
$$(q_1, a) \rightarrow q_3$$

$$(q_1, \epsilon) \rightarrow q_2$$

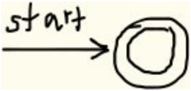


a
1
ε
start
0
a
ε
2
b

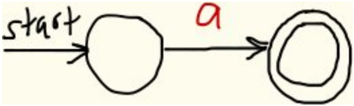
$\varnothing$



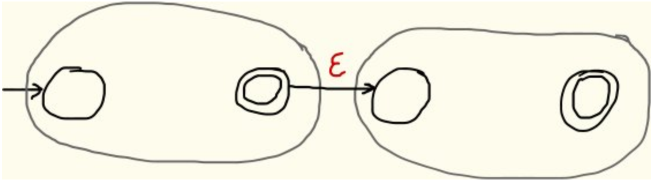
ϵ

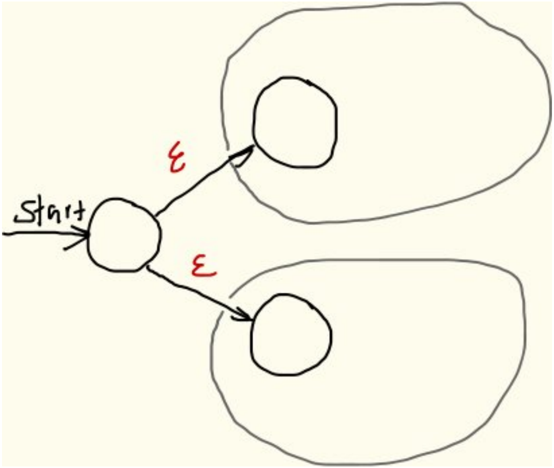


c

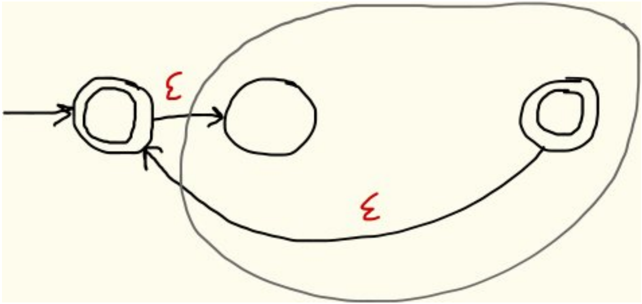


$\mathbf{r}_1 \cdot \mathbf{r}_2$



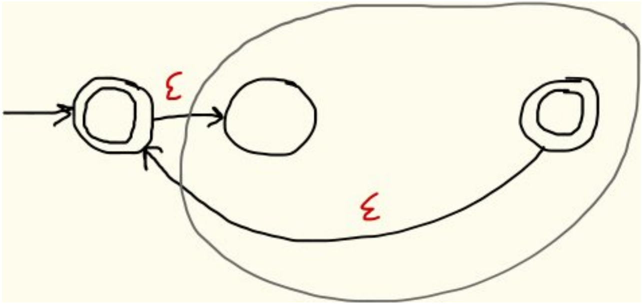


$\mathbf{r}_1 + \mathbf{r}_2$



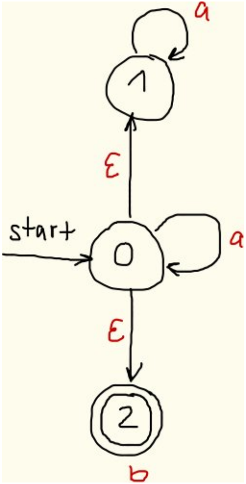
r^*

r*



Why can't we just have an epsilon transition from the accepting states to the starting state?

Subset Construction



	a	b
$\varnothing$	$\varnothing$	$\varnothing$
$\{0\}$	$\{0,1,2\}$	$\{2\}$
$\{1\}$	$\{1\}$	$\varnothing$
$\{2\}$	$\varnothing$	$\{2\}$
$\{0,1\}$	$\{0,1,2\}$	$\{2\}$
$\{0,2\}$	$\{0,1,2\}$	$\{2\}$
$\{1,2\}$	$\{1\}$	$\{2\}$
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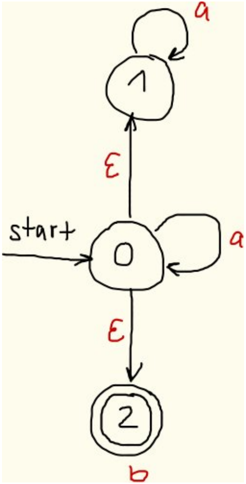

	a	b
$\varnothing$	$\varnothing$	$\varnothing$
$\{0\}$	$\{0, 1, 2\}$	$\{2\}$
$\{1\}$	$\{1\}$	$\varnothing$
$\{2\}$ *	$\varnothing$	$\{2\}$
$\{0, 1\}$	$\{0, 1, 2\}$	$\{2\}$
$\{0, 2\}$ *	$\{0, 1, 2\}$	$\{2\}$
$\{1, 2\}$ *	$\{1\}$	$\{2\}$
s: $\{0, 1, 2\}$ *	$\{0, 1, 2\}$	$\{2\}$

Regular Languages

A language is regular iff there exists a regular expression that recognises all its strings.

or equivalently

A language is regular iff there exists a deterministic finite automaton that recognises all its strings.

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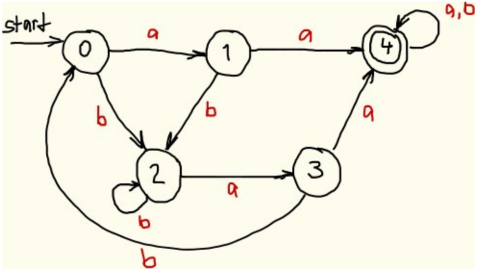
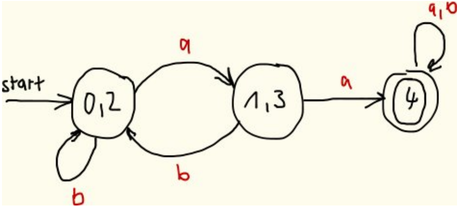
Why is every finite set of strings a regular language?





minimal automaton

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   are marked. If yes, then also mark (q, p)
4. Repeat last step until no chance.
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Given the function

$$
\begin{aligned}
\boldsymbol{rev}(\varnothing) &\stackrel{\text{def}}{=} \varnothing \\
\boldsymbol{rev}(\boldsymbol{\epsilon}) &\stackrel{\text{def}}{=} \boldsymbol{\epsilon} \\
\boldsymbol{rev}(\boldsymbol{c}) &\stackrel{\text{def}}{=} \boldsymbol{c} \\
\boldsymbol{rev}(\boldsymbol{r}_1 + \boldsymbol{r}_2) &\stackrel{\text{def}}{=} \boldsymbol{rev}(\boldsymbol{r}_1) + \boldsymbol{rev}(\boldsymbol{r}_2) \\
\boldsymbol{rev}(\boldsymbol{r}_1 \cdot \boldsymbol{r}_2) &\stackrel{\text{def}}{=} \boldsymbol{rev}(\boldsymbol{r}_2) \cdot \boldsymbol{rev}(\boldsymbol{r}_1) \\
\boldsymbol{rev}(\boldsymbol{r}^*) &\stackrel{\text{def}}{=} \boldsymbol{rev}(\boldsymbol{r})^*
\end{aligned}
$$

and the set

$$\boldsymbol{Rev}\ \boldsymbol{A} \stackrel{\text{def}}{=} \{\boldsymbol{s}^{-1} \mid \boldsymbol{s} \in \boldsymbol{A}\}$$

prove whether

$$\boldsymbol{L}(\boldsymbol{rev}(\boldsymbol{r})) = \boldsymbol{Rev}(\boldsymbol{L}(\boldsymbol{r}))$$

- The star-case in our proof about the matcher needs the following lemma

$$\text{Der}\,c\,A^* = (\text{Der}\,c\,A)\,@\,A^*$$

- If $'''' \in A$, then
$\text{Der}\,c\,(A\,@\,B) = (\text{Der}\,c\,A)\,@\,B \cup (\text{Der}\,c\,B)$

- If $'''' \notin A$, then
$\text{Der}\,c\,(A\,@\,B) = (\text{Der}\,c\,A)\,@\,B$

- Assuming you have the alphabet $\{a, b, c\}$
- Give a regular expression that can recognise all strings that have at least one b.

"I hate coding. I do not want to look at code."