Homework 2

1. Review the first handout about sets of strings and read the second handout. Assuming the alphabet is $\{a, b\}$, decide which of the following equations are true in general for arbitrary languages A, B and C:

$$\begin{array}{rcl} (A \cup B)@C & = & A@C \cup B@C \\ A^* \cup B^* & = & (A \cup B)^* \\ A^*@A^* & = & A^* \\ (A \cap B)@C & = & (A@C) \cap (B@C) \end{array}$$

   In case an equation is true, give an explanation; otherwise give a counterexample.

2. What is the meaning of a regular expression? Give an inductive definition.

3. Given the regular expressions $r_1 = \epsilon$ and $r_2 = \varnothing$ and $r_3 = a$. How many strings can the regular expressions r_1^*, r_2^* and r_3^* each match?

4. Give regular expressions for (a) decimal numbers and for (b) binary numbers. (Hint: Observe that the empty string is not a number. Also observe that leading 0s are normally not written.)

5. Decide whether the following two regular expressions are equivalent $(\epsilon + a)^* \equiv^? a^*$ and $(a \cdot b)^* \cdot a \equiv^? a \cdot (b \cdot a)^*$.

6. Given the regular expression $r = (a \cdot b + b)^*$. Compute what the derivative of r is with respect to a and b. Is r nullable?

7. Prove that for all regular expressions r we have

$$\text{nullable}(r) \quad \text{if and only if} \quad \texttt{""} \in L(r)$$