Compilers and Formal Languages

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1 Introduction, Languages
2 Regular Expressions, Derivatives
3 Automata, Regular Languages
4 Lexing, Tokenising
5 Grammars, Parsing
6 While-Language
7 Compilation, JVM
8 Compiling Functional Languages
9 Optimisations
10 LLVM

Coursework 1: Submissions

- Scala (162)
- Ocaml (1)
- Java (1) …uses new features of Java 21
- Rust (6)

Parser


lexer

parser

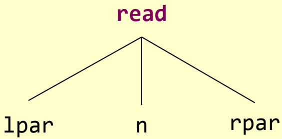
code gen

Parser

parser input: a sequence of tokens

```
key(read) lpar id(n) rpar semi
```

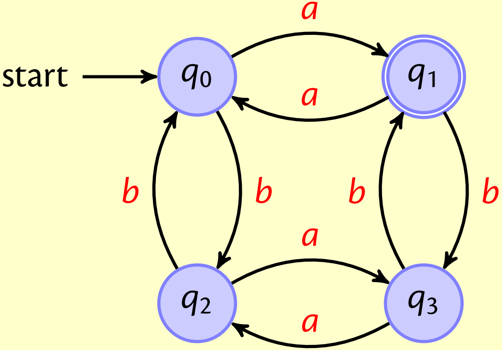
parser output: an abstract syntax tree



What Parsing is Not

Usually parsing does not check semantic correctness, e.g.

- whether a function is not used before it is defined
- whether a function has the correct number of arguments or are of correct type
- whether a variable can be declared twice in a scope



Which language?

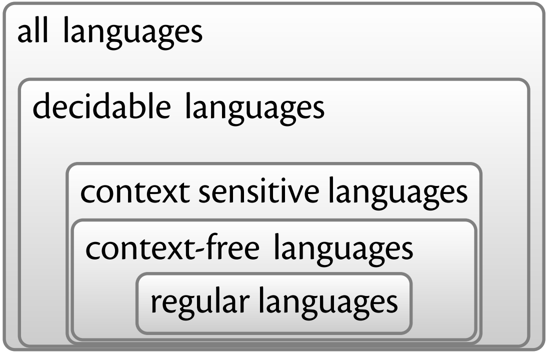
Regular Languages

While regular expressions are very useful for lexing, there is no regular expression that can recognise the language $a^n b^n$.

$$(((()()))()) \quad \text{vs.} \quad (((()()))()))$$

So we cannot find out with regular expressions whether parentheses are matched or unmatched. Also regular expressions are not recursive, e.g. $(1 + 2) + 3$.

Hierarchy of Languages



all languages

decidable languages

context sensitive languages

context-free languages

regular languages

Time flies like an arrow.
Fruit flies like bananas.

CFGs

A **context-free grammar** G consists of

- a finite set of nonterminal symbols (e.g. A upper case)
- a finite set terminal symbols or tokens (lower case)
- a start symbol (which must be a nonterminal)
- a set of rules

$$A ::= rhs$$

where rhs are sequences involving terminals and nonterminals, including the empty sequence ϵ.

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where *rhs* are sequences involving terminals and nonterminals, including the empty sequence ϵ.

We also allow rules

$$A ::= rhs_1 | rhs_2 | \ldots$$

Palindromes

A grammar for palindromes over the alphabet $\{a, b\}$:

$$S ::= a \cdot S \cdot a$$

$$S ::= b \cdot S \cdot b$$

$$S ::= a$$

$$S ::= b$$

$$S ::= \epsilon$$

Palindromes

A grammar for palindromes over the alphabet $\{a, b\}$:

$$\mathbf{S} ::= a \cdot \mathbf{S} \cdot a \mid b \cdot \mathbf{S} \cdot b \mid a \mid b \mid \epsilon$$

Arithmetic Expressions

$$
\begin{aligned}
E ::= {} & 0 \mid 1 \mid 2 \mid \ldots \mid 9 \\
\mid {} & E \cdot + \cdot E \\
\mid {} & E \cdot - \cdot E \\
\mid {} & E \cdot * \cdot E \\
\mid {} & (\cdot E \cdot)
\end{aligned}
$$

Arithmetic Expressions

$$
\begin{array}{rcl}
E & ::= & 0 \mid 1 \mid 2 \mid \ldots \mid 9 \\
  & \mid & E \cdot + \cdot E \\
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  & \mid & (\cdot E \cdot)
\end{array}
$$

`1 + 2 * 3 + 4`

A CFG Derivation

1. Begin with a string containing only the start symbol, say S
2. Replace any nonterminal X in the string by the right-hand side of some production $X ::= rhs$
3. Repeat 2 until there are no nonterminals left

$$S \rightarrow \ldots \rightarrow \ldots \rightarrow \ldots \rightarrow \ldots$$

Example Derivation

$$S ::= \epsilon \mid a \cdot S \cdot a \mid b \cdot S \cdot b$$

$$\begin{array}{rcl} S & \rightarrow & aSa \\ & \rightarrow & abSba \\ & \rightarrow & abaSaba \\ & \rightarrow & abaaba \end{array}$$

Example Derivation

$$
\begin{array}{rcl}
E & ::= & 0 \mid 1 \mid 2 \mid \ldots \mid 9 \\
  & \mid & E \cdot + \cdot E \\
  & \mid & E \cdot - \cdot E \\
  & \mid & E \cdot * \cdot E \\
  & \mid & (\cdot E \cdot)
\end{array}
$$

$$
\begin{array}{ll}
E \rightarrow & E * E \\
\rightarrow & E + E * E \\
\rightarrow & E + E * E + E \\
\rightarrow^{+} & 1 + 2 * 3 + 4
\end{array}
$$

Example Derivation

$$\begin{array}{rl} E ::= & 0 \mid 1 \mid 2 \mid \ldots \mid 9 \\ & \mid E \cdot + \cdot E \\ & \mid E \cdot - \cdot E \\ & \mid E \cdot * \cdot E \\ & \mid (\cdot E \cdot) \end{array}$$

$$\begin{array}{rl} E \rightarrow & E * E \\ \rightarrow & E + E * E \\ \rightarrow & E + E * E + E \\ \rightarrow^{+} & 1 + 2 * 3 + 4 \end{array} \qquad \begin{array}{rl} E \rightarrow & E + E \\ \rightarrow & E + E + E \\ \rightarrow & E + E * E + E \\ \rightarrow^{+} & 1 + 2 * 3 + 4 \end{array}$$

Language of a CFG

Let G be a context-free grammar with start symbol $\mathbf{S}$. Then the language $L(G)$ is:

$$\{c_1 \ldots c_n \mid \forall i.\, c_i \in T \wedge \mathbf{S} \rightarrow^* c_1 \ldots c_n\}$$

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$$\{c_1 \ldots c_n \mid \forall i.\ c_i \in T \wedge \mathbf{S} \rightarrow^* c_1 \ldots c_n\}$$

- Terminals, because there are no rules for replacing them.
- Once generated, terminals are "permanent".
- Terminals ought to be tokens of the language (but can also be strings).

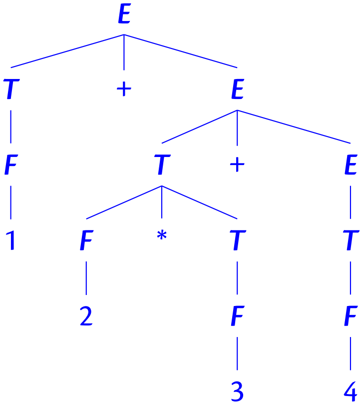
Parse Trees

$$E ::= T \mid T \cdot + \cdot E \mid T \cdot - \cdot E$$

$$T ::= F \mid F \cdot * \cdot T$$

$$F ::= 0...9 \mid (\cdot E \cdot)$$

1 + 2 * 3 + 4



Arithmetic Expressions

$$
\begin{array}{rcl}
E & ::= & 0..9 \\
  & | & E \cdot + \cdot E \\
  & | & E \cdot - \cdot E \\
  & | & E \cdot * \cdot E \\
  & | & (\cdot E \cdot)
\end{array}
$$

Arithmetic Expressions

$$\begin{aligned} E ::= {} & 0..9 \\ \mid {} & E \cdot + \cdot E \\ \mid {} & E \cdot - \cdot E \\ \mid {} & E \cdot * \cdot E \\ \mid {} & (\cdot E \cdot) \end{aligned}$$

A CFG is **left-recursive** if it has a nonterminal E such that $E \rightarrow^{+} E \cdot \ldots$

Ambiguous Grammars

A grammar is **ambiguous** if there is a string that has at least two different parse trees.

$$
\begin{aligned}
E ::= {} & 0 \ldots 9 \\
| {} & E \cdot + \cdot E \\
| {} & E \cdot - \cdot E \\
| {} & E \cdot * \cdot E \\
| {} & (\cdot E \cdot)
\end{aligned}
$$

`1 + 2 * 3 + 4`

‘Dangling’ Else

Another ambiguous grammar:

$$\begin{array}{lcl} E & \rightarrow & \textit{if } E \textit{ then } E \\ & \mid & \textit{if } E \textit{ then } E \textit{ else } E \\ & \mid & \ldots \end{array}$$

```
if a then if x then y else c
```

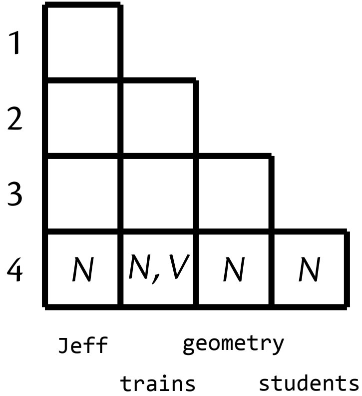
CYK Algorithm

Suppose the grammar:

$$\begin{array}{lcl} S & ::= & N \cdot P \\ P & ::= & V \cdot N \\ N & ::= & N \cdot N \\ N & ::= & \texttt{students} \mid \texttt{Jeff} \mid \texttt{geometry} \mid \texttt{trains} \\ V & ::= & \texttt{trains} \end{array}$$

`Jeff trains geometry students`

CYK Algorithm



1				
2				
3				
4	N	N, V	N	N
	`Jeff`	`trains`	`geometry`	`students`

$$
\begin{array}{lcl}
S & ::= & N \cdot P \\
P & ::= & V \cdot N \\
N & ::= & N \cdot N \\
N & ::= & \texttt{students} \mid \texttt{Jeff} \\
  &   & \mid \texttt{geometry} \mid \texttt{trains} \\
V & ::= & \texttt{trains}
\end{array}
$$

Chomsky Normal Form

A grammar for palindromes over the alphabet $\{a, b\}$:

$$\mathbf{S} ::= a \cdot \mathbf{S} \cdot a \mid b \cdot \mathbf{S} \cdot b \mid a \cdot a \mid b \cdot b \mid a \mid b$$

CYK Algorithm

- fastest possible algorithm for recognition problem
- runtime is $O(n^3)$

- grammars need to be transformed into CNF

> *"The C++ grammar is ambiguous, context-dependent and potentially requires infinite lookahead to resolve some ambiguities."*

from the PhD thesis by Willink (2001)

```
int(x), y, *const z;
int(x), y, new int;
```

Context Sensitive Grammars

It is much harder to find out whether a string is parsed by a context sensitive grammar:

$$\begin{array}{rcl} S & ::= & bSAA \mid \epsilon \\ A & ::= & a \\ bA & ::= & Ab \end{array}$$

Context Sensitive Grammars

It is much harder to find out whether a string is parsed by a context sensitive grammar:

$$\begin{aligned} S &::= bSAA \mid \epsilon \\ A &::= a \\ bA &::= Ab \end{aligned}$$

$$S \rightarrow \ldots \rightarrow^{?} ababaa$$

For CW2, please include '\' as a symbol in strings, because the collatz program contains

```
write "\n";
```

val (r1s, f1s) = simp(r1)
val (r2s, f2s) = simp(r2)
how are the first rectification functions f1s and f2s made? could you maybe show an example?

Questions regarding CFL CW1

Dear Dr Urban

Regarding CW1, I am stuck on finding the nullable and derivative rules for some important regexes.

The NOT Regex nullable rule: I am not sure how to approach this, I am inclined to simply put this as the negation of the nullable function on the input regex (e.g !nullable(r)). However I have found instances where negating a nullable does not make it un-nullable. For example the negation of r* can still match regex ab (which is not nullable). So I would like some actual clarification, pointers and help in this area.

The NOT Regex derivation rule: again I am dumbfounded here, I am inclined to think that I should derive the regex and then negate that derivation. But none of this ever works. Please provide some helpful information so I can solve this.